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Precise measurement of renal filtration and vascular parameters using a two-compartment model for dynamic contrast-enhanced MRI of the kidney gives realistic normal values

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Abstract

Objective To model the uptake phase of T₁-weighted DCE-MRI data in normal kidneys and to demonstrate that the fitted physiological parameters correlate with published normal values.

Methods The model incorporates delay and broadening of the arterial vascular peak as it appears in the capillary bed, two distinct compartments for renal intravascular and extravascular Gd tracer, and uses a small-vessel haematocrit value of 24%. Four physiological parameters can be estimated: regional filtration K^{trans} (ml min⁻¹ [ml tissue]⁻¹), perfusion F (ml min⁻¹ [100 ml tissue]⁻¹), blood volume v_b

(%) and mean residence time MRT (s). From these are found the filtration fraction (FF ; %) and total GFR (ml min⁻¹). Fifteen healthy volunteers were imaged twice using oblique coronal slices every 2.5 s to determine the reproducibility. **Results** Using parenchymal ROIs, group mean values for renal biomarkers all agreed with published values: K^{trans} : 0.25; F : 219; v_b : 34; MRT: 5.5; FF : 15; GFR: 115. Nominally cortical ROIs consistently underestimated total filtration (by ~50%). Reproducibility was 7–18%. Sensitivity analysis showed that these fitted parameters are most vulnerable to errors in the fixed parameters kidney T₁, flip angle, haematocrit and relaxivity.

Conclusions These renal biomarkers can potentially measure renal physiology in diagnosis and treatment.

Key Points

- Dynamic contrast-enhanced magnetic resonance imaging can measure renal function.
- Filtration and perfusion values in healthy volunteers agree with published normal values.
- Precision measured in healthy volunteers is between 7 and 15%.

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Keywords DCE-MRI · Kidney · GFR · Quantification · Modeling

Introduction

Dynamic imaging of renal uptake of a contrast agent is an established way of assessing renal physiology using nuclear medicine, dynamic computed tomography and magnetic resonance imaging (MRI), and estimation of quantitative parameters is possible [1–8]. Several reviews are available [9–11].

Dynamic contrast-enhanced (DCE) MRI of the kidneys is now clinically feasible with the advent of fast sequences.

This paper builds on the compartmental modelling approach that has been so successful in characterising capillary leakage in tumours [12, 13], with the addition of a delay and broadening of the arterial vascular peak as is observed in the kidney. This is most likely caused by a small delay along the renal artery and a non-zero residence time in the renal capillary bed. The renal model, applicable to cortical and parenchymal ROIs, captures the essential features of the dynamic data, yet remains simple.

The published modelling work in normal subjects generally shows little systematic effort to reconcile the measurements with published values from other (non-MRI) methods, and there are few systematic measurements of precision (repeatability). Very often DCE images are just reported visually (i.e. qualitatively). In this quantitative study, testing the new model using data from healthy volunteers shows that key renal physiological parameters related to filtration and perfusion are estimated with good reproducibility and give measurements close to published normal values. Preliminary versions of this work have been presented orally [14–16]. This paper should be read with the Electronic Supplementary Material (ESM).

Materials and methods

Pharmacokinetic and MRI model

The model consists of two parts (full mathematical details are given in the Appendix). The pharmacokinetic part (see Fig. 1 and Eqs. 1–5 below) is applicable to the cortex and parenchyma. It describes the intra-renal concentrations of

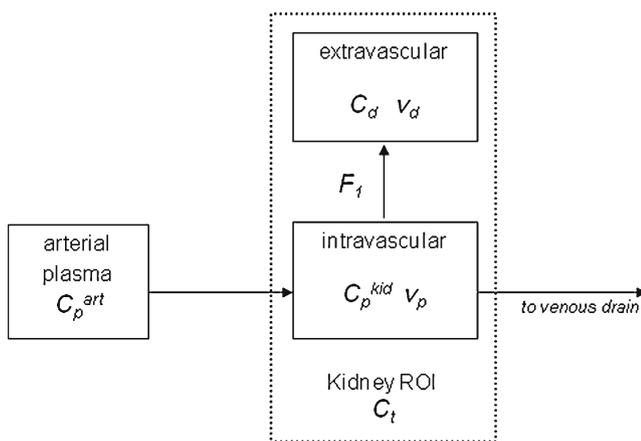


Fig. 1 Two-compartment model for renal filtration. The intravascular (IV) compartment is primarily glomerular; the extravascular (EV) compartment is primarily tubular. The dotted box represents an ROI or pixel in the kidney. This model is used soon after Gd bolus arrival, before there is time for efflux from the EV compartment

intravascular (IV; largely glomerular) and extravascular (EV; mostly tubular) Gd contrast agent (also called tracer) in renal tissue (Fig. 1). The arterial vascular peak is delayed and broadened (dispersed) before arrival at the IV compartment (the delay is apparent in the data, see Fig. 3a below). This process is described by a vascular impulse response function (VIRF); thus, the IV plasma concentration in the kidney is the convolution of the arterial plasma concentration with the VIRF (Eq. 1 below). The VIRF is the response in the renal vasculature to a very short pulse (in fact a mathematical delta function) of Gd in the aorta (see Fig. 6 below). From the VIRF renal perfusion F (ml blood min^{-1} (100 ml tissue) $^{-1}$) was estimated. Uptake or flux from the IV to the EV compartment (by the process of filtration, whereby Gd is removed from capillaries) is $F_f = K^{trans} C_p^{kid}(t)$ per unit volume of tissue, where K^{trans} is the regional filtration (GFR per unit volume of kidney; formally the volume transfer constant [12] from plasma), $C_p^{kid}(t)$ is the time-varying tracer plasma concentration in the IV compartment, and F_f has units of mmol min^{-1} (ml tissue) $^{-1}$. This transportation of Gd into the ROI by vascular means is appropriate in a cortical or parenchymal ROI, but not in a medullary ROI.

It is known [4, 17] that no filtered tracer leaves the kidney for a certain time after arrival in the blood capillaries (if a parenchymal region is used, this time is usually about 90 s); the model is used to analyse data over this relatively short time period, and therefore it can be assumed that there is no efflux from the EV compartment.

The MRI part of the model (Eqs. 6–9) defines how Gd concentration enhances the MRI signal. It relies on knowledge of several fixed parameter values (see Table 1 below): T_1 of blood and kidney, Gd relaxivity, haematocrit, and the imaging parameters TR and flip angle FA.

In the fitting procedure, the model is adjusted by varying the free parameters in the model until the difference between

Table 1 Filtration values and fit residuals from the uptake phase of repeated imaging of 15 healthy volunteers

ROI	VIRF	K^{trans} (min^{-1})			RMS residual
		Mean	SD ^a	ISD ^b	(Mean; %)
Cortical	Delayed exponential	0.29	0.058	0.046	3.9
	Gaussian	0.30	0.060	0.046	4.8
Parenchymal	Delayed exponential	0.25	0.058	0.043	4.0
	Gaussian	0.25	0.059	0.043	4.5

VIRF = vascular impulse response function, ISD = instrumental standard deviation, RMS = root-mean-square

^a Group SD

^b Instrumental SD (from repeated imaging)

the model and the measured signal data is minimised. The free parameters are the filtration K^{trans} , the blood volume v_b , and the VIRF parameters (one for broadening, and if necessary a second parameter to describe delay). Thus the instant exponential VIRF was defined by only one free parameter, whilst the delayed exponential and Gaussian VIRFs required two free parameters (see Appendix). For each fit the RMS (root-mean-square) residual (i.e. difference between a data point and the model value) was found. The local filtration fraction is the ratio of regional filtration to regional renal plasma flow, and was calculated using Eq. 10.

MRI acquisition and analysis

Fifteen healthy volunteers were imaged twice (about 13 days apart) at 1.5 T with a temporal resolution of 2.5 s. A half-dose of Gd-DPTA was used. No T_1 values were measured. Image datasets were registered to remove in-plane movement [18]. Regions of interest (ROIs) were placed over the aorta and kidney. A parenchymal ROI was defined semi-automatically on the perfusion image as proposed by Peters et al. [19]. An ROI of the whole cortex was drawn manually (Fig. 2). Full details of MRI acquisition and ROI generation are given in the ESM.

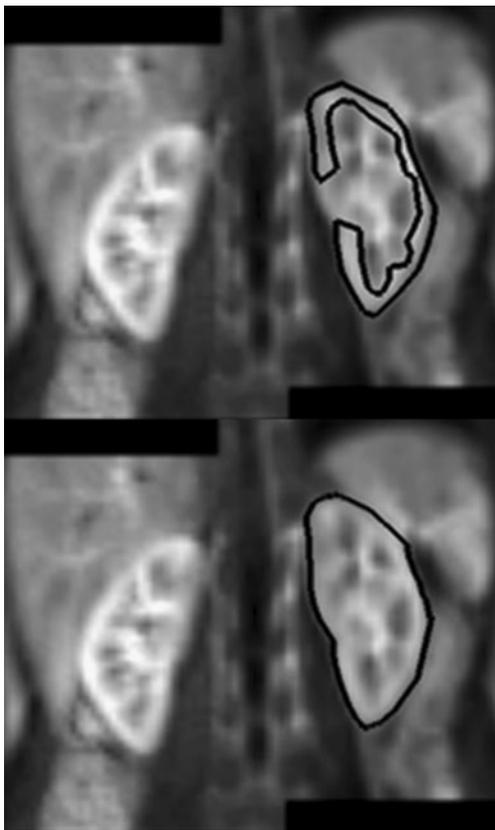


Fig. 2 Cortical and parenchymal ROIs

In the model analysis, two phases (i.e. time periods) of data were fitted: (1) the perfusion phase (up to the kidney signal minimum after the first bolus passage) when Gd is largely IV, and (2) the uptake (filtration) phase (up to 90 s after bolus arrival, before any Gd has left the parenchymal ROI). Renal filtration and vascular parameters (including local filtration, filtration fraction, perfusion, blood volume and mean residence time) were estimated. We hypothesised that vascular parameters might be better estimated from perfusion phase data than from uptake phase data; fits were compared on the basis of reproducibility and residuals.

Three plausible VIRFs (instant exponential, delayed exponential and Gaussian) were compared in cortical and parenchymal ROIs on the basis of the quality of the model fit and the reproducibility of perfusion estimates. The values of the seven fixed tissue parameters (see Table 5) were initially set at: $Hct^{large} = Hct^{small} = 41\%$ [20]; $r_1^{blood} = r_1^d = r_1^{iv} = 4.5s^{-1}mM^{-1}$ (the in vitro value [21]); $T_{10}^{blood} = 1.4s$ [22]; $T_{10}^{kidney} = 1.2s$ (average of cortical and medullary values [23]).

Instrumental SD (ISD) For each kidney, the differences in repeated measures of model parameters were calculated, to give the standard deviation in a single measurement, using the method of Bland and Altman [24]. In this approach, the differences between repeated measurements in the same subject are found; the standard deviation of these differences are calculated and then divided by 1.4 to find the standard deviation in a single measurement [25].

Total filtration The filtration parameter found by the model (GFR per unit volume of tissue, or K^{trans}) is an intensive quantity (thus density and temperature are intensive quantities as they do not automatically increase with size, as mass does). A relationship can be established between K^{trans} and GFR, which is the quantity used clinically. From the mean K^{trans} in a ROI, the total filtration in that ROI can be found ($GFR_{ROI} = K^{trans} \times ROI \text{ volume}$). As increasingly large ROIs are used, we expect to see a plateau in GFR_{ROI} , as all of the functioning kidney in that slice is included. ROIs of increasing size were created manually, from a small piece of cortex up to ROIs that were larger than ('over included') all of the parenchyma in a slice. (Here 'over-inclusion' means that all the partial-volume pixels that could possibly be part of the parenchyma were included.) This object strength approach overcomes partial volume effects and the difficulty of drawing precise ROIs, and has been used to characterise objects with indistinct borders [26]. Eight kidney datasets were analysed (2 subjects, each with repeated imaging).

Model sensitivity analysis The vulnerability of the various tissue parameter estimates (K^{trans} etc., see Table 5 below) to

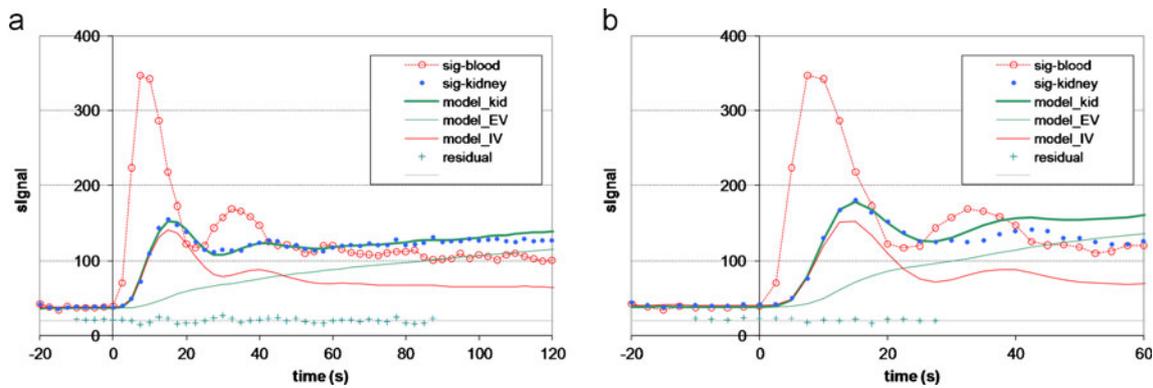


Fig. 3 Example fits of model to data from a single healthy kidney, showing two phases of data, with delayed Gaussian vascular impulse response function (VIRF). **a** Parenchymal ROI, uptake phase; **b** cortical ROI, perfusion phase. Residuals are shown for fitting period only

(vertically offset in the plot, for clarity). Note evidence of efflux after the end of the fitting period (data dip below model). Separate IV and EV contributions to the model are shown

error in the fixed parameters (r_1 , T_{10} etc.) was found by calculating the error propagation ratio (EPR) [27]. In this sensitivity analysis, the model is used to determine by what percentage a tissue parameter estimate will change as a result of a 1% error in a fixed parameter. For example a +1% error in T_{10}^{kid} will cause an approximately -1% error in K^{trans} ; thus the EPR from T_{10}^{kid} to K^{trans} is -1.0.

Normal values for kidney filtration and vascular parameters were calculated from our measurements, and compared with other (non-MRI) normal values from the literature (see Table 4). Total kidney volume was calculated from the pre-Gd T₁-weighted images for the 15 normal subjects. Values standardised to a body surface area (BSA) of 1.73 m² were also found; BSA was estimated from $BSA(m^2) = 0.0235 \text{ height}(cm)^{0.422} \text{ weight}(kg)^{0.514}$ [28].

Results

Typical fits for the two phases of data are shown in Fig. 3. Sixty kidney image datasets were each fitted in two phases and with three VIRFs, giving a total of 360 fitting operations. The cubic interpolation of the aortic blood data gives a convincing description between the data points (Fig. 4) and enables timing parameters (Δ , T_g , T_{fwhm} , MRT) to be calculated with precision well below that of the imaging temporal resolution. Delayed exponential and Gaussian VIRFs fitted well. The onset of efflux from both the cortical and parenchymal regions could be detected (see for example Fig. 3a).

Comparing the three VIRFs in terms of the residuals from fits, for cortical and parenchymal regions in the perfusion phase, it was clear that the instant exponential VIRF performed significantly less well (RMS residual 4.3–6.5%) than the delayed exponential and Gaussian VIRFs (residuals 2.6–3.6%). Fractional blood volume v_b mean values and reproducibility (from the repeated examinations) were

approximately the same for all VIRFs, for cortical and parenchymal ROIs and in both perfusion and uptake phases. The instant exponential VIRF was therefore excluded from subsequent analysis.

Comparing perfusion and uptake phases (for the estimation of v_b) showed that reproducibility was always slightly better using uptake phase data, for both cortical and parenchymal ROIs (although the residuals from the fits were slightly higher in the uptake phase). The hypothesis that the perfusion phase would estimate perfusion parameters better than the uptake phase, and justify the extra complexity of carrying out two fits, was therefore disproved, and perfusion phase data were not further analysed.

Measured normal values, residuals and measurement precision (ISD) are summarised in Tables 1 and 2.

Total filtration GFR values for increasingly large ROIs reached a plateau (Fig. 5). The datasets analysed showed similar behaviour, although some had a less obvious

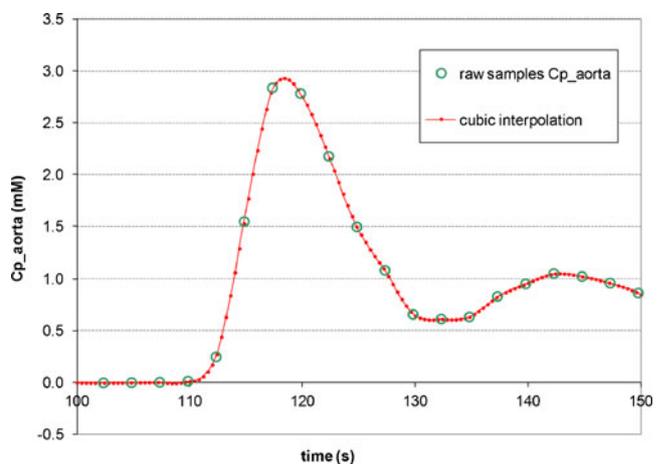


Fig. 4 Estimated arterial plasma concentration from blood signal, using cubic interpolation; note detail in peak not present in raw data

Table 2 Perfusion parameters from the uptake phase of repeated imaging of 15 healthy volunteers. Residuals are shown in Table 1 and symbols in Table 5

MRT = mean residence time

^aFrom peak and MRT of delayed exponential or Gaussian VIRF; units: ml blood min⁻¹ (100 ml tissue)⁻¹

^bGroup SD

^cInstrumental SD (from repeated imaging)

v_b (%)			MRT (s)			$F^{peak\ a}$			$F^{MRT\ a}$		
Mean	SD ^b	ISD ^c	Mean	SD	ISD	Mean	SD	ISD	Mean	SD	ISD
Cortical ROI, VIRF = delayed exponential											
41	10	7	5.20	0.66	0.35	542	150	104	482	127	77
Cortical ROI, VIRF = Gaussian											
40	10	8	4.68	0.68	0.37	310	80	44	523	138	71
Parenchymal ROI, VIRF = delayed exponential											
45	12	8	5.89	0.67	0.48	477	164	68	465	141	66
Parenchymal ROI, VIRF = Gaussian											
44	11	8	5.49	0.73	0.40	284	89	71	495	156	42

plateau, with GFR_ROI increasing beyond the nominal parenchymal outline. The plateau value was 11.2 ± 2.8 ml min⁻¹ (mean \pm SD; 2 subjects, 8 kidneys). Typical values for a GFR_ROI calculation in a nominally cortical ROI were: $K^{trans} = 0.23$ min⁻¹; ROI size = 361 pixels; voxel volume 73 mm³; GFR_ROI = 6.1 ml min⁻¹. The effective number of slices per kidney was 5.8 (taking into account that peripheral slices contain smaller volumes of kidney); thus the parenchymal data give an estimate for total GFR of about 130 ml min⁻¹ by this method, in agreement with other methods (see Table 4).

Sensitivity analysis is shown in Table 3. This confirms the large influence that the fixed parameters haematocrit, relaxivity, T_{10} and FA have on the estimated values of tissue parameters (principally filtration, blood volume and perfusion). Mean residence time is unaffected by fixed parameter errors, whilst the filtration fraction is more robust than filtration or perfusion. Small deviations in fixed parameters were used (~ 1 –3%) to avoid non-linear effects, and the resulting EPR values had small random errors ($\sim 1\%$); thus a measured EPR of 0.99 could in fact be 1.00.

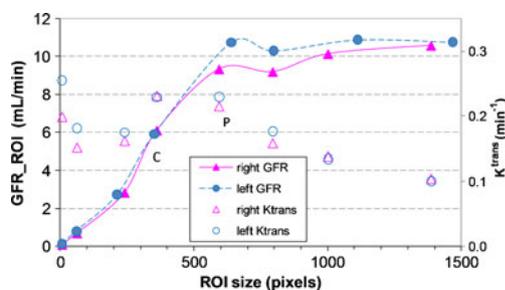


Fig. 5 In a central kidney slice, glomerular filtration rate (GFR) in progressively larger ROIs increases to a plateau value of about 11 ml min⁻¹; for five slices per kidney this gives a total GFR of 110 ml min⁻¹ (single kidney GFR=55). Nominal cortical (C=350 pixels) and parenchymal (P=600 pixels) ROIs are shown. Mean K^{trans} values for small cortical ROIs vary, then decrease progressively for ROIs larger than the cortex

Normal values for kidney filtration and vascular parameters, measured using other (non-MRI) methods, were taken from the literature (see Table 4), and our measurements were compared with these. Kidney volume V_{kid} was estimated from a published value of mass m_{kid} as follows. The mass was measured post-mortem and therefore excludes most of the blood (which would drain out after removal). If a fraction of blood α remains in the parenchyma, then $V_{kid} = \frac{m_{kid}}{\rho(1-(1-\alpha)v_b)}$, where ρ is the kidney density. Using $m_{kid} = 150$ g [29], $\rho = 1.03$ g ml⁻¹, $\alpha = 10\%$, $v_b = 35\%$ gives $V_{kid} = 213$ ml, close to a published value of 218 ml [30].

The small vessel haematocrit value Hct^{small} is much lower than Hct^{large} (red blood cells have difficulty entering

Table 3 Sensitivity analysis showing how fitted (free) parameters (K^{trans} etc.) are affected by chosen value of fixed parameter (e.g. Hct^{large}). Values are error propagation ratio (EPR; i.e. percentage change in fitted parameter for 1% change in fixed parameter) [27]. Symbols are defined in Table 5

	$K^{trans\ b}$	v_b^a	F^b	MRT	FF
Fixed tissue parameters					
Hct^{large}	-0.72	-0.69	-0.70	0	0
Hct^{small}	0	+0.69	+0.69	0	0
r_l^{blood}	+0.98	+1.03	+1.03	0	0
r_l^d	-0.97	0	0	0	-0.99
r_l^{iv}	0	-0.98	-0.98	0	+0.99
$r_l^{iv} = r_l^{blood}$	+0.99 ^c	0	0	0	+0.99
T_{10}^{blood}	+1.04	+1.24	+1.29	0.02	-0.22
T_{10}^{kidney}	-1.06	-1.08	-1.14	0	+0.22
Fixed instrumental parameters					
θ	-0.07	+0.25	+0.29	0.03	-0.38
TR	0	-0.13	-0.15	0	+0.16

^a Average from delayed exponential and Gaussian VIRFs

^b Average of 4 F values from delayed exponential and Gaussian VIRFs, using both peak and MRT (see Eqs. 15, 16, 18, 19 in *ESM*)

^c r_l^{iv} fixed = r_l^{blood} ; both altered together

Table 4 Comparison of normal parenchymal renal parameters estimated using DCE-MRI with published values. Thirty normal kidneys were each measured twice, using the uptake phase and Gaussian VIRF.

Using a lower value for small-vessel haematocrit dramatically reduces values for blood volume (right-hand column)

		MRI $Hct^{small}=41\%$ Mean (SD)	Instrumental SD (CV)	Literature value	MRI $Hct^{small}=24\%$ Mean (SD)
Filtration (min^{-1})	K^{trans}	0.25 (0.05)	0.04 (18%)	0.28 ^a	0.25 (0.05)
Mean residence time (s)	MRT	5.5 (0.7)	0.4 (7%)	6.5 ^b	5.5 (0.7)
Blood volume (%)	v_b	44 (11)	8 (18%)	35 ^c	34 (8)
Perfusion ^d ml blood min^{-1} (100 ml tissue) ⁻¹	F	284 (89)	72 (14%) ^e	264 ^f	219 (67)
Filtration fraction (%) ^g	FF	15.5 (2.9)	1.5 (9%)	15–20 ^h	15.5 (2.9)
Absolute single kidney volume (ml)	V_{kid}	230 (28)	Not measured	213 ⁱ	230 (28)
Standardised single kidney volume (ml) ^j	V_{kid}^*	214 (20)	Not measured	213 ⁱ	214 (20)
Total GFR (ml min^{-1})	GFR	115 (27) ^k	Not measured	125 ^h	115 (27) ^k

CV Coefficient of Variation

^a $\text{GFR}/(2V_{kid}^*)$ ^b Measured using MRI by Sourbron [6]: ‘plasma transit time TP’; SD=1.3 s^c From CT [43]^d From peak of Gaussian VIRF F_{gauss}^{peak} ; average perfusion over parenchymal ROI; plasma perfusion is independent of Hct^{small} (see text)^e ISD of cortical perfusion is better (14%; see Table 2)^f Mean parenchymal perfusion = $\text{RBF}/2 * V_{kid}$; $\text{RBF}=1.1 \text{ l min}^{-1}$ [1, 46]^g Using Eq. 10^h Typical for young adult males [46]ⁱ Estimated using mass=150 g (see text)^j I.e. corrected for body surface area (see text)^k $2 K^{trans} V_{kid}$

small channels); values of 24% (dog heart) [31], 31% (human brain) [32], 25% [33] and 8–20% [34] have been reported. This is related to the Fahraeus effect; in small vessels red blood cells travel faster than plasma [34, 35]. The renal vasa recta (10–20 μm in diameter) have a reduced haematocrit of 40–50% compared with a large vessel [36]; the network Fahraeus effect can further reduce this by as much as 20% [34].

The MRI values were recalculated using a value of 24% for Hct^{small} (Table 4, right hand column); blood volume values then agreed with literature values. Left-sided cortical values of v_b and F were significantly higher than right-sided values ($P < 10^{-8}$).

The effect on our normal values of altering the fixed tissue parameters revealed the following. Reducing T_{10}^{kid} from 1.2 s to 1.1 s (a plausible value for a parenchymal ROI dominated by cortical uptake; see Discussion in ESM) increased values of K^{trans} , v_b and F by about 9%, whilst leaving FF unaltered. Reducing r_1^d towards the low values indicated by rat studies [37] (e.g. $2.0 \text{ s}^{-1} \text{ mM}^{-1}$; see Discussion in ESM) gave unrealistically high values of K^{trans} (0.56 min^{-1}), whilst leaving v_b , F and FF unaltered.

Discussion

A much fuller discussion is given in the Electronic Supplementary Material (ESM).

Imaging biomarkers and renal function

The kidney has many functions in maintaining homeostasis, one of which, glomerular filtration, is critical in both clinical nephrology and kidney research. The kidney’s filtration fraction (FF) is defined as the ratio of GFR to renal plasma flow (RPF); however, RPF is difficult to measure (both experimentally and clinically). The mathematical model presented in this paper provides parameters that can give quantitative single kidney renal perfusion and GFR values as well as measurements of blood volume and FF (Table 4).

Existing MRI techniques

Various models [1–11] have been proposed for analysis of renal filtration using DCE-MRI. A critical review [38] showed that none of the published methods was sufficiently

accurate to be clinically accepted. The Patlak model and graphical analysis approach have been proposed for renal MRI [4, 17, 39–41], and comparisons have been made [1, 5] with more modern models.

Computed tomography

In DCE-CT imaging [41–43] the relationship between tracer concentration and signal intensity is linear. However partial volume effects and the estimation of small vessel haematocrit are problems for both CT and MRI.

Benefits of this model

Analysis of the early (uptake) phase of DCE-MRI using this model measures a few critical biomarkers of kidney function, namely vascular parameters and filtration. By restricting the time period over which the data are analysed (i.e. excluding efflux of tracer from the ROI), a simpler model can be used, designed specifically to measure vascular parameters and filtration. By analysing parenchymal (not cortical) ROIs, this analysis period has been extended to 90 s. This gives a time window where the domination of signal behaviour by filtration can be exploited using an optimal model, which is simple and therefore precise. Only a single free parameter is required to characterise renal function. The model represents the complexity of the actual kidney DCE-MRI signal data (Fig. 3) whilst avoiding undue further complexity; this probably contributes to being able to estimate the model parameters reliably [44]. This is an example of Occam's razor, which states that if a variety of explanations of a phenomenon are available, then in the absence of any other information, the simplest one is to be preferred.

Two additional kidney parameters are produced in this analysis. The filtration fraction (Eq. 10) is a valuable parameter in diabetes and other clinical diseases. The mean

residence time (MRT) is very stable (Table 4), and its physiological significance needs to be evaluated. Although the MRT is measured from the aortic ROI, transit along the renal artery is rapid [45].

Shortcomings of DCE-MRI measurements

Uncertainty in the haematocrit value affects some of the tissue parameters and a realistic value for small vessel haematocrit (Hct^{small}) is required. The relationship between renal blood and plasma flow is not straightforwardly given by the large vessel haematocrit (as implied in text books [46]), and CT measurements would be equally affected. The higher values in the left-sided kidneys are probably an artefact, for which a possible explanation is poor slice profile [47].

Tissue parameter estimates are vulnerable to errors in haematocrit, tissue relaxivity, T_{10} and flip angle (Table 3). Estimates of renal plasma flow F_p (and also plasma volume v_p) are independent of Hct^{small} , and F_p might be a more useful tissue parameter than F . Our normal parenchymal values (from Table 4) are $F_p=167$ ml plasma min^{-1} (100 ml tissue) $^{-1}$, $v_p=26\%$.

Relaxivity can alter in vivo from the in vitro values [37] and is probably the largest source of systematic error in DCE-MRI studies, as well as being unavoidable [48–50]. In disease, T_{10} is often raised, and should be measured explicitly if possible. In the absence of a measurement, then published values must be used [23, 51, 52]. Flip angle errors are likely at 3 T; a simple B_1 mapping technique [53] takes only a few minutes.

Cortical and parenchymal ROIs For estimation of filtration the entire parenchyma needs to be included in the ROI as the filtrate progresses relentlessly down the tubules into the medulla and back up to the cortex. Thus, the nominally cortical ROIs seriously underestimate uptake (Fig. 5). The

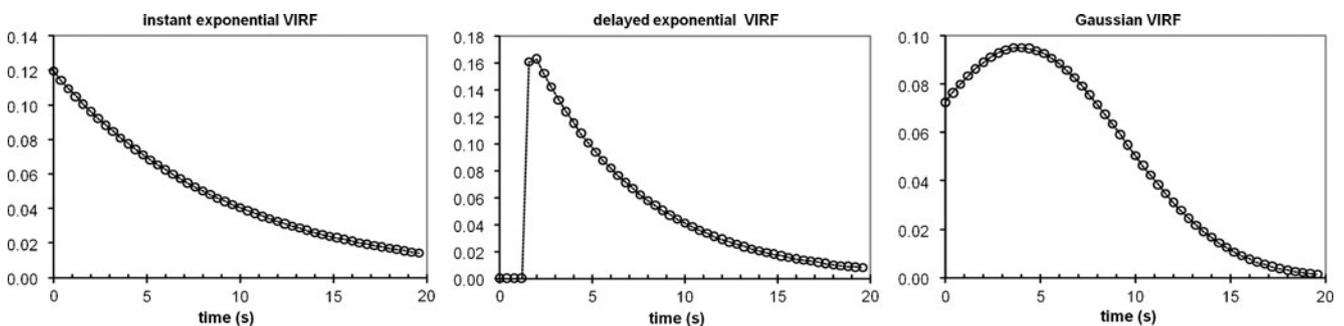


Fig. 6 Vascular impulse response functions (VIRFs). All fitted the data shown in Fig. 3a (parenchymal ROI, uptake phase) and have unit area. The instant exponential VIRF modelled the delayed perfusion

peak badly. Differing peak values give rather different estimates for perfusion (see Table 2), although mean residence times are similar for both delayed VIRFs

Table 5 Parameters used for modelling

Quantity	Symbol	Units	Type
Concentration in aortic blood	C_b	mM	
Concentration in extravascular space	C_d	mM	
Concentration in aortic plasma	C_p^{aorta}	mM	
Concentration in kidney plasma	C_p^{kid}	mM	
Concentration in kidney tissue	C_t	mM	
VIRF delay	Δ	s	Free
Perfusion	F	ml blood (100 ml tissue) ⁻¹ min ⁻¹	
Filtration fraction	FF	%	
Flow into extravascular space per unit volume of tissue	F_1	mmol s ⁻¹ ml ⁻¹	
Flow out of extravascular space per unit volume of tissue ^a	F_2	mmol s ⁻¹ ml ⁻¹	
Flip angle	θ	Degrees	Fixed (17°)
Haematocrit in large vessels	Hct^{large}	%	Fixed (41%)
Haematocrit in capillaries	Hct^{small}	%	Fixed (41% or 24%)
Filtration (= GFR per unit volume of tissue)	K^{trans}	min ⁻¹ b	Free
Mean residence time ^c	MRT	s	
T ₁ relaxivity in blood	r_1^{blood}	s ⁻¹ mM ⁻¹	Fixed (4.5 s ⁻¹ mM ⁻¹) [21]
T ₁ relaxivity in extravascular space	r_1^d	s ⁻¹ mM ⁻¹	Fixed (4.5 s ⁻¹ mM ⁻¹)
T ₁ relaxivity in intravascular space	r_1^{iv}	s ⁻¹ mM ⁻¹	Fixed (4.5 s ⁻¹ mM ⁻¹)
Pre-Gd blood signal	$S(0)^{blood}$	A.U.	Pre-calculated ^d
Pre-Gd tissue signal	$S(0)^{kidney}$	A.U.	Pre-calculated ^d
T ₁ of blood	T_{10}^{blood}	s	Fixed (1.4 s)
T ₁ of kidney	T_{10}^{kidney}	s	Fixed (1.2 s)
Gaussian VIRF width	T_{fwhm}	s	Free
Exponential VIRF width	T_g	s	Free
TR	TR	s	Fixed (1.6 ms)
Fractional blood volume in kidney	v_b	0 < v_b < 1	Free
Fractional plasma volume in kidney	v_p	0 < v_p < 1	

^a Used for model with efflux (Eqs. 20, 21)

^b Or can be expressed in ml min⁻¹ (100 ml tissue)⁻¹ to be compatible with F

^c From VIRF

^d For each dataset, found from pre-Gd blood and tissue signals (arbitrary units)

variable ROI definition is probably a major contributor to within-subject variation [54].

VIRF shapes None of the VIRFs used in this work (see Fig. 6) is completely satisfactory; this is unsurprising given the complexity of renal anatomy, and the precise form is unimportant for estimation of filtration (and hence GFR) or blood volume (Table 1). An instant exponential model of the renal VIRF does not fit the vascular peak as well as a delayed exponential or Gaussian, although it has been used in the two-compartment exchange model (2CXM) [55]. The 2CXM may be too simple a model for kidney vasculature. Work is in progress to identify appropriate VIRFs.

The model has been extended to cover efflux, by adding a single extra free parameter [14] (see [ESM](#)).

Clinical studies will establish whether the DCE-MRI parameters will be sensitive to alterations in disease state such as focal renal damage in reflux nephropathy, disease states that affect the kidneys asymmetrically (obstruction or stone disease) or following renal transplantation. If these DCE-MRI biomarkers are shown to be reproducible then the clinician will have a non-invasive tool that avoids any radiation burden, and a quick single test will provide both anatomy and physiological parameters of each kidney separately. Recent publications show the advantage of quantitative renal perfusion measurements over conventional MRA [56, 57]. Our study has established the feasibility of using our model to measure both blood flow parameters and filtration with good reproducibility and reasonable accuracy in normal volunteers (Table 4).

Conclusion

Our model will calculate perfusion, filtration, filtration fraction, mean residence time, blood volume and single kidney GFR. It has been simplified by using uptake–phase time-domain data from parenchymal ROIs, it deals with signal non-linearity, it uses a reasonably realistic vascular impulse response function, and it recognises a reduced value of small vessel haematocrit. This model now needs to be evaluated on datasets from other centres, and in patients.

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Appendix: Two-compartment mathematical model with broadening and delay of vascular peak

A1 Mathematical model (Fig. 1)

The perfusion peak (first pass of the bolus) in the kidney tissue data is consistently delayed and broadened compared with that in the arterial blood curve (see Fig. 2). Here the intravascular plasma concentration in the kidney $C_p^{kid}(t)$ is modelled as a convolution of the arterial plasma concentration $C_p^{art}(t)$ with a simple normalised vascular impulse response function (VIRF) $g(t)$:

$$C_p^{kid}(t) = C_p^{art}(t) \otimes g(t) = \int_0^t C_p^{art}(t - \tau)g(\tau)d\tau \tag{1}$$

$$\int_0^\infty g(t)dt = 1 \tag{2}$$

Thus $g(t)$ is the IV response to an arterial delta function. Depletion of IV Gd by filtration is assumed to be small (i.e. $FF \ll 1$). The VIRF gives a variable delay and broadening. Several functions for $g(t)$ were investigated (see below).

The rate of uptake F_I into the renal extravascular space (filtration) is proportional to the IV concentration:

$$v_d \frac{dC_d(t)}{dt} = F_I = K^{trans} C_p^{kid}(t) \tag{3}$$

v_d is the fractional volume of the renal EV space ($0 < v_d < 1$), $C_d(t)$ is the time-dependent concentration in this space, and F_I is the flow rate of Gd into the EV compartment. K^{trans} here is the filtration (GFR) per unit volume of tissue; it is formally the unidirectional local transfer constant [12] for

Gd from the IV space. The solution for $C_d(t)$ is a simple integral:

$$v_d C_d(t) = K^{trans} \int_0^t C_p^{kid}(\tau)d\tau \tag{4}$$

The concentration of IV tracer in tissue is $v_p C_p^{kid}$, where the fractional volume of the IV plasma space $v_p = v_b(1 - Hct^{small})$; v_b is the fractional blood volume in the kidney and Hct^{small} is the haematocrit in small vessels such as capillaries. The total Gd tissue concentration $C_t(t)$ is then the sum of the IV and EV contributions (which we have in Eqs. 1 and 4):

$$C_t(t) = v_b(1 - Hct^{small})C_p^{kid} + v_d C_d(t) \tag{5}$$

The MRI signal enhancement from a given Gd concentration $C(t)$ in blood or tissue is straightforward (assuming fast exchange for water, so that all the water in a voxel is relaxed by all the Gd). The reduction in T_1 is given by:

$$R_1(t) = R_{I0} + r_1 C(t) \tag{6}$$

R_I is the relaxation rate ($R_I = 1/T_I$), R_{I0} ($= 1/T_{I0}$) is its native (i.e. pre-Gd) value (before injection of contrast agent) and r_I is the relaxivity (change in relaxation rate per unit concentration of Gd). The possibility of differing relaxivities in the IV space, the EV space and the arterial blood can be incorporated into fuller versions of this equation for tissue and blood:

$$\begin{aligned} R_1^{kidney}(t) &= R_{I0}^{kidney} + r_1^{iv} v_b(1 - Hct^{small})C_p^{kid}(t) + r_1^d v_d C_d(t) \\ R_1^{blood}(t) &= R_{I0}^{blood} + r_1^{blood} C_b(t) \end{aligned} \tag{7}$$

where individual values r_1^{iv} , r_1^d , r_1^{blood} are used for each compartment (see Table 5).

The signal from a spoiled gradient echo sequence is:

$$S(t) = S_0 \frac{(1 - e^{-R_1(t)TR}) \sin \theta}{1 - e^{-R_1(t)TR} \cos \theta} \tag{8}$$

where θ is the flip angle.

The Gd concentration in the artery $C_p^{art}(t)$ (required for Eq. 1) is obtained from the measured blood signal as follows. Given $S(0)^{blood}$ (measured before the arrival of Gd) and T_{I0}^{blood} we can find S_0^{blood} (from Eq. 8). From the observed blood signal (time–intensity curve) $S(t)^{blood}$ can then be found $R_1^{blood}(t)$ (Eq. 8). We can find the Gd concentration in arterial blood $C_b^{art}(t)$ (using Eq. 7). The concentration in plasma is then related by:

$$C_b^{art}(t) = (1 - Hct^{large})C_p^{art}(t) \tag{9}$$

where Hct^{large} is the haematocrit in arteries.

The curve-fitting procedure, the discrete representation of the continuous functions at a temporal resolution of 0.4 s, spreadsheet implementation and interpolation of the AIF using Everett's formula for cubic interpolation [58] are all described in the [ESM](#).

A2: Vascular impulse response functions: estimation of perfusion and filtration fraction

Three VIRFs were implemented as discrete functions (see [ESM](#)), forced to zero for $t < 0$ and normalised to have a unit area over their finite duration (up to about 20 s), i.e. $\sum_i g_i(\tau) d\tau = 1$ (Fig. 6). For each, the mean residence time (MRT) was found from the first moment of $g(t)$.

Perfusion F was estimated from the VIRF peak or MRT. The filtration fraction is then simply the ratio of GFR to renal plasma flow $F(1-Hct^{small})$:

$$FF = \frac{K^{trans}}{(1 - Hct^{small})F} \quad (10)$$

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