Magnetic Resonance in Medicine 50:649–653 (2003)

Removing Spikes Caused by Quantization Noise From High-Resolution Histograms

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A novel method is presented for the removal of spikes, caused by the division of two series of integers, from high-resolution histograms. When two series of integers are divided the results take the form of a nonuniform distribution. Such a division is often used in medical imaging, due to the storage of most images as integers. An example of this is the division of the saturated and unsaturated signal intensities to obtain a magnetization transfer ratio. Histograms produced using these methods often contain spikes relating to the nonuniform distribution mentioned above. These spikes can have serious implications for certain histogram characteristics. Most commonly, peak height and location can be seriously distorted by these spikes, which have predictable locations. These spikes can be removed by the addition of uniformly distributed noise to the integer signal intensities before division. Magn Reson Med 50:649–653, 2003. © 2003 Wiley-Liss, Inc.

Key words: histograms; spike removal; quantization noise; high resolution

Histogram analysis is an established tool for the interpretation of images in MRI. Histograms have been used with a variety of parameters such as the magnetization transfer ratio (MTR) (1) and mean diffusivity (2). Many analysis methods are used, ranging from simple measures such as peak height and location (3), to more complicated analysis such as principal component analysis (4). The importance of smooth histograms for all methods of histogram interpretation cannot be underestimated; simple parameters such as the peak height or location can be completely altered by unwanted spikes, while for more complex analysis these spikes could cause subtle changes in the results which may go unnoticed by the investigator.

One characteristic often in conflict with smoothness is the resolution of the histogram. Most often the choice of bin width is decided by the units of the variable plotted. For example T_1 histograms are often plotted in units of 1 ms and MTR histograms in units of 0.1 pu (percentage units, as described in Ref. 5). Although this is appropriate in many cases, under- or overresolution in histograms can affect the results obtained from them.

Histograms spikes come from a variety of sources, such as noise contamination of the image data and systematic errors in the image acquisition or processing methods. Spikes are traditionally removed by the application of a moving average window, or median filter, to the data, which smoothes the histogram. Unfortunately, use of a window that is too wide or on histograms containing "real" narrow spikes can result in a loss of information.

This work looks at the removal of spikes that occur when a histogram is formed by the division of two sets of integers, in particular, the case when the result of the division is <1. It is shown that this division causes spikes to appear in T_1 and MTR histograms and that the addition of uniformly distributed noise with a mean of zero and a range of ± 0.5 to the integers prior to division removes these spikes without degrading the data.

The cause of these spikes will be described in the Theory section; however, briefly, they are caused by the natural abundance of fractions such as 1/2, 2/3, and 3/4 when two integers are divided. The spikes take the form of a high point in the histogram and a neighboring low point. Two examples of this are shown in Fig. 1; both histograms are whole-head, as the effect is independent of tissue. First a T_1 histogram is shown which was derived using the method described by Parker et al. (6). A lookup table relates the ratio of the signal intensities of T_1- to PD-weighted images to a specific T_1 value. As can be seen from the graph, there are a number of spikes and corresponding troughs throughout the histogram. Some spikes consist of one high point, such as in the right-hand tail of the histogram, and some of several points, such as seen at values surrounding 1130 ms. The increased width of some of these spikes is due to the variation in flip angle and RF field inhomogeneity across the sample. The T_1 calculation program takes note of these, meaning that the T_1 of a pixel is not only a function of the ratio of the two signal intensities, but also the calculated flip angle. So a given ratio of T_1-weighted signal intensity to PD-weighted signal intensity corresponds to a number of T_1 values giving the width in the spikes. The second example is an MTR histogram; the use of a division in the calculation of these histograms is more obvious and, again, a similar problem can be seen. Artifacts appear as single points raised above the surrounding histogram, accompanied by a corresponding low point.

The spikes in these histograms occur when the ratio of the two signal intensities in question is equal to values such as 0.5, 0.25, 0.286 (=2/7), and 0.167 (=1/6) for the T_1 histogram and MTR values of 50, 25, and 66.7 pu, to name but a few. These histograms are good examples of the problems caused by this issue. The peak height of the MTR histogram is 4.85% volume/pu; however, this occurs at a value of 37.5 pu (=3/8) and appears to be slightly to the right of where it should occur, suggesting that it may not be an accurate estimate of the peak height or peak location. If this is indeed due to the integer division problem described here, then the measured peak location for all MTR

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Grant sponsor: Multiple Sclerosis Society of Great Britain and Northern Ireland; Grant number: 491/I193398.

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Received 4 February 2003; revised 15 April 2003; accepted 22 April 2003.
DOI 10.1002/mrm.10561
Published online in Wiley InterScience (www.interscience.wiley.com).

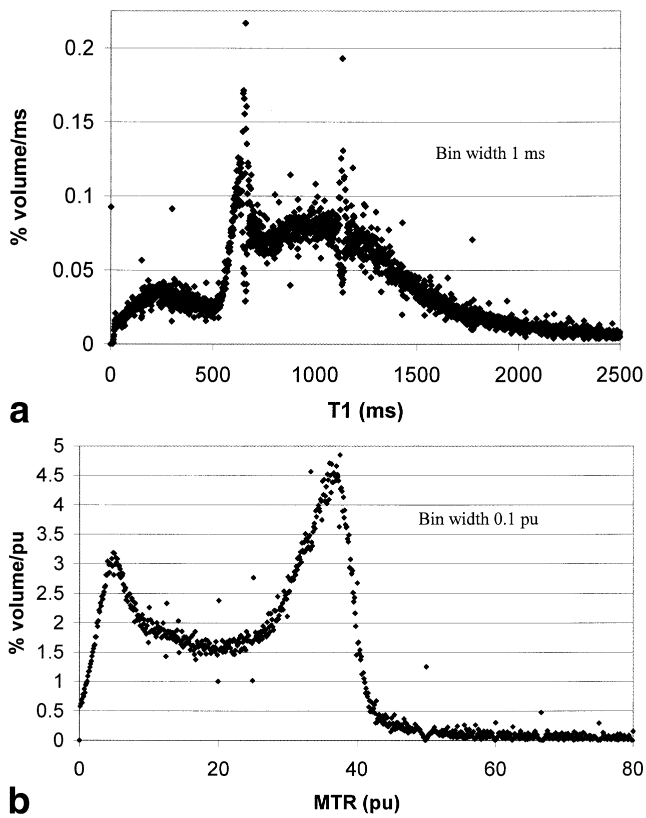

FIG. 1. **a:** T_1 histogram, bin width 1 ms, calculated using the method described by Parker et al. (6). Note the spikes at around 700 and 1100 ms; although the first of these has an anatomical basis, it appears elongated and slightly offset by the artifact, the second spike appears to be totally artifactual. Note also the singular spikes and their corresponding troughs that occur throughout the histogram. **b:** MTR histogram, bin width 0.1 pu. Again, many points are offset from the main curve of the histogram.

histograms may occur at this value, whereas in fact the true peak may be subtly shifted between two groups of subjects. Similarly, the main peak height in the T_1 histogram at 658 ms does not appear to be realistic compared with the heights of surrounding T_1 values. It appears that the spike is artificially high, as is another value at 1134 ms. Indeed, this value is nearly as high as the main spike itself, so a small variation in the histogram could again cause errors in the estimation of peak location as well as peak height. The height of the artificial spikes is a combination of the natural frequency of the specific ratio, as will be discussed below, and the true height of the histogram at that point. This explains why the artifactual spikes seem to have the greatest impact at areas of high intensity in the histogram, even though the ratio associated with these points does not have a large natural frequency.

THEORY

The division of two integers does not necessarily result in a third integer; this is always the case when the numerator is less than the denominator and greater than zero. The distribution of the resulting fractions is of importance when calculating histograms where the parameter under investigation involves the division of two signal intensities. This is because signal intensities from a pixel are stored as an integer after scaling by most, if not all, image analysis programs and by the scanner equipment itself. The other technical factor affecting the distribution of these numbers is the limited dynamic range of the scanning hardware; the maximum signal intensity seen in systems with 12 bit data storage is 4096 arbitrary units (au) and for most images the actual range used extends only to ~1500 au.

The natural frequency of a ratio of two integers comes from the distribution of the fractional results of the division a/b, where b takes each integer value between 1 and the maximum intensity value in an image and a loops between 1 and b-1 for each value of b. A plot of this distribution (not shown) shows that it is uneven and that a clear pattern can be seen. Most values occur an approximately equal number of times in the distribution; however, there are a number of spikes. The highest of these corresponds to a ratio of 1/2, 2 lower spikes occur at 1/3 and 2/3, then two more at 1/4 and 3/4, and so on. This result can be easily explained mathematically. For every second integer there is another integer which has half its value; however, only every third integer has an integer with a third of its value, and so on. It should also be noted that for each high point in the distribution there is a corresponding neighboring low point where the value of the histogram is reduced, often to zero.

The problems caused by this distribution are increased as the bin size is reduced. For larger bins the increased bin width can blur this distribution to the point where the artifactual spikes become insignificant; however, as the bin size is reduced the relative difference in peak heights increases due to the discontinuities in the resultant quotients. This is shown in Fig. 2, where data simulating a MTR histogram are plotted with a variety of bin sizes. Two Gaussians of mean 1000, SD 100 (g(y)) and mean 600, SD 100 (h(z)) were used to represent the unsaturated and saturated images, respectively. The two distributions are divided $(\text{MTR}(y, z) = 100 \times (g(y)-h(z))/g(y))$ and histograms produced for a variety of bin widths. Figure 2a clearly shows the expected spikes at "MTRs" including 50 pu and 40 pu; in Fig. 2b these have been reduced in size and some have completely disappeared; in Fig. 2c only remnants of the spikes at 40 and 50 pu are visible as slight steps in the histogram.

The origin of these histogram spikes is the discontinuous representation of the source distributions (divisor and dividend) by integers. This is known as quantization noise and occurs when analog signals are converted to digital values such as in many MRI systems. In a floating point representation these distributions consist of delta functions at integer values and zeros elsewhere. By replacing these with a continuous distribution extending half-way between neighboring spikes, it is expected that the appearance of the spikes will be drastically reduced. Although this problem manifests itself in medical imaging due to the use of integers to store image data, it is not confined to this, but can occur whenever the histogram bin size is smaller in magnitude than the precision of the numbers divided to create a quotient. Storing the image in floating point format would solve the problem in most examples, as the preci-

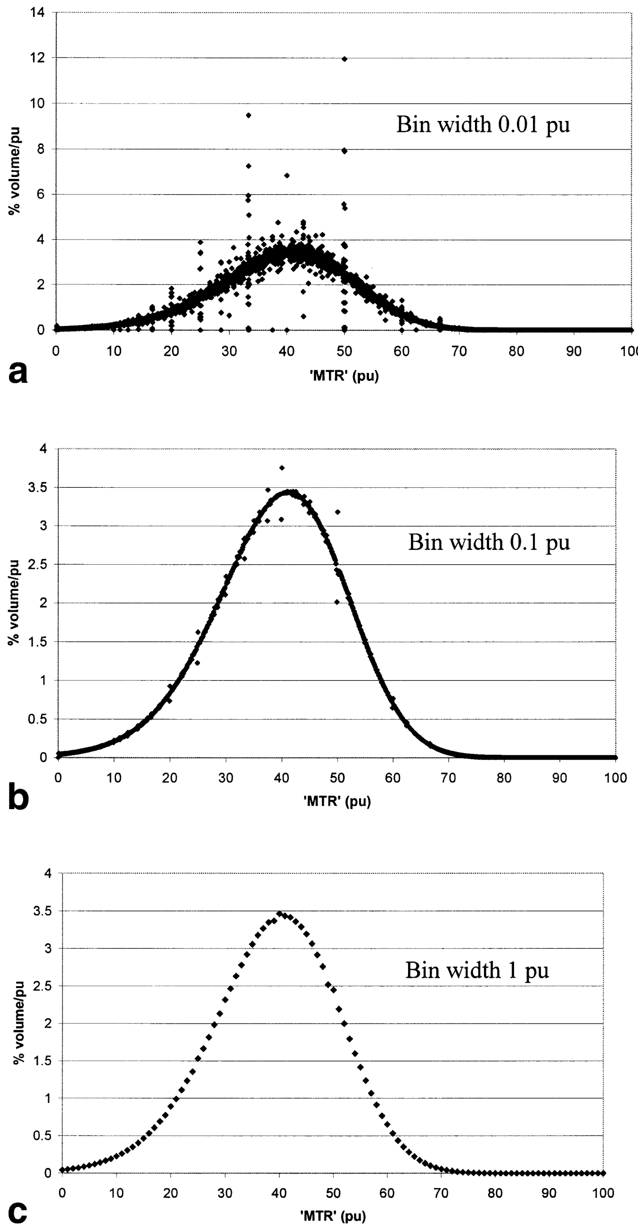

FIG. 2. The data shown in these histograms were obtained from the division of two Gaussian distributions to simulate an MTR histogram. The bin width is 0.01, 0.1, and 1 pu for (**a**), (**b**), and (**c**), respectively. It can be seen that the histograms become smoother as the bin width increases.

sion of the data could be made smaller than the bin width. However, given that the random noise in an image is likely to be an order of magnitude greater than the rounding error caused by integer data representation, the cost in technical and data storage terms would probably exceed the benefit from such a move.

MATERIALS AND METHODS

To remove these spikes noise was added to the integer signal intensities. Three noise addition schemes were tested. In the first of these the noise was uniformly distributed with a mean of 0 and a range of ±0.5. The second scheme added normally distributed noise with a mean of zero and SD of ±0.5. The last scheme involved estimating the gradient of the histograms of the two input images as a linear function for each signal intensity and using this as the probability distribution function for the addition of noise with the same range as above at each point with this signal intensity. The addition of uniformly distributed noise was chosen as it should most closely resemble the rounding which occurs during the analog-to-digital conversion performed by the scanner, so its effect on the data should be of a similar magnitude to this. A smaller effect should be seen when normally distributed noise is used, as the noise value is likely to be smaller in magnitude, minimizing the difference between the original and corrected signal intensities. By comparing the effect of these schemes and the third noise addition scheme, which has less scientific basis behind it and should be reasonably different from the other two, the effect of the choice of noise addition scheme can be assessed and compared with the effect of adding noise at all.

Although it is true that the numbers produced are not random in a strict definition, it is accepted that a good random number generator is adequate for most applications, certainly one as simple as this (7,8). It is not suggested that this technique replaces any smoothing that is appropriate but that it is performed prior to smoothing to allow a better determination of the true histogram. To test whether smoothing has a similar effect and whether the two techniques can be used in parallel, T_1 histograms were calculated with and without the correction which were then smoothed with a window size of 7 ms.

Example pseudo-code for this procedure and the calculation of MTR histograms is given below.

```
/*IO and IS are arrays containing the integer signal intensities of pixels in the unsaturated and saturated images, respectively, noise and noise1 are arrays containing series of noise values as described above. SO and SS are the floating point signal intensities with the noise added to them*/
Integer IO[], IS[], x_dim, y_dim, z_dim
Float SO, SS, noise[], noise1[], MTR
get_uniform_noise(x_dim*y_dim*z_dim, noise) /*fill array noise with noise*/
get_uniform_noise(x_dim*y_dim*z_dim, noise1) /*fill array noise1 with noise*/
for(i=0; i<= x_dim*y_dim*z_dim; i++)
SO = float(IO)+noise
SS = float(IS)+noise1
MTR(i) = 1000*(SO-SS)/SO
End for
```

RESULTS

The three noise addition schemes were tested on a number of both T_1 and MTR image sets. The histogram mean, peak height, peak location, and the 25th and 75th centile points were compared and it was found that there was no significant difference between the metrics produced by the three schemes; all three removed the artifactual spikes and the histogram parameters varied only slightly, as would be

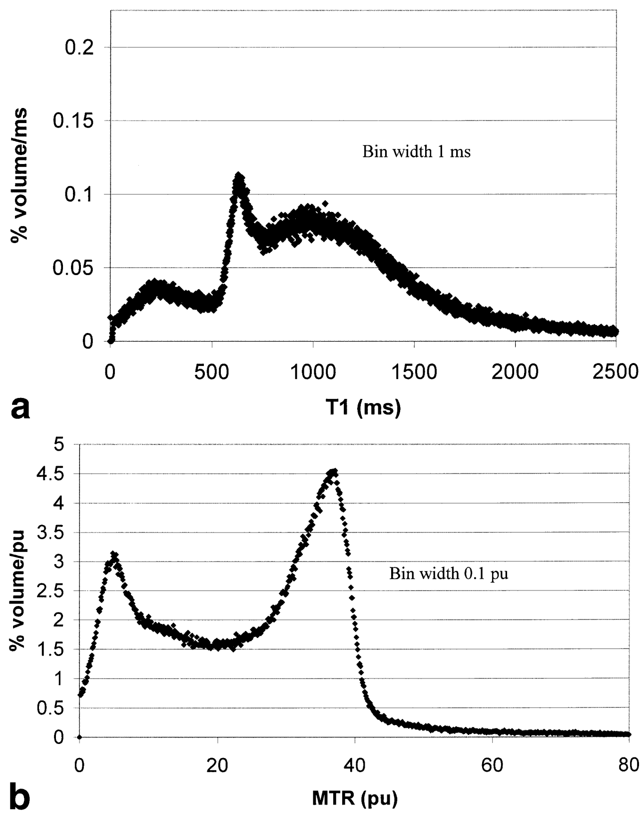

FIG. 3. The corrected histograms. **a:** Corresponds to Fig. 1a but with the noise correction applied. The spikes have disappeared while the rest of the histogram is largely unaffected. Interestingly, the peak height has changed dramatically due to the correction. A similar result is seen in **b**, the MTR histogram. The artifactual high and low points are gone and the histogram takes on a much smother appearance.

expected, due to the slight variations in the noise added signal intensities. Visual inspection of the histograms produced also indicated that there were no systematic differences between the three schemes. The histograms shown here are those produced by the addition of uniformly distributed noise.

Figure 3 shows histograms of the same data as presented in Fig. 1, but with noise added as described above. In these histograms the artifactual spikes relating to the distribution described have been removed and the histogram takes on a much smoother appearance. In the MTR histogram it can be seen that the artificial peak at 37.5 pu has been removed and the true spike occurs at 37.1 pu with a peak height of 4.55% volume/pu. Also, the high point at 50 pu and the other spikes in the histogram have been removed and the corresponding low points are increased to more believable values. The overall shape of the histogram does not seem to have altered at all and there appear to be no major differences in peak height for any points other than those that are clearly artifactual. Similar results are achieved on the T_1 histogram, where it can be seen that the peak height is reduced from 0.22% volume/ms in the uncorrected histogram to 0.11% volume/ms, a value which seems more consistent with the rest of the histogram, compared with the large value seen in the uncorrected histogram. The peak location also seems to have a more realistic value of 622 ms compared with 658 ms. While the value itself is believable, the position of 658 ms does not appear, on close examination, to quite fit with the dynamics of the histogram shape—it is a little too far to the right. The secondary spike at 1134 ms is also removed, again leaving the histogram with a much smoother appearance.

Analysis of the residuals shows that there is little change in the rest of the histogram. There are random differences in the height of each bin, as would be expected; however, these are small in magnitude and apart from the removal of the spikes there is no systematic deviation between the histograms, indicating that there is no degradation in the information that can be obtained from them.

Figure 4 shows the T_1 histogram smoothed with a median filter of size 7. The lower of the two histograms is derived from images which have had the noise correction,

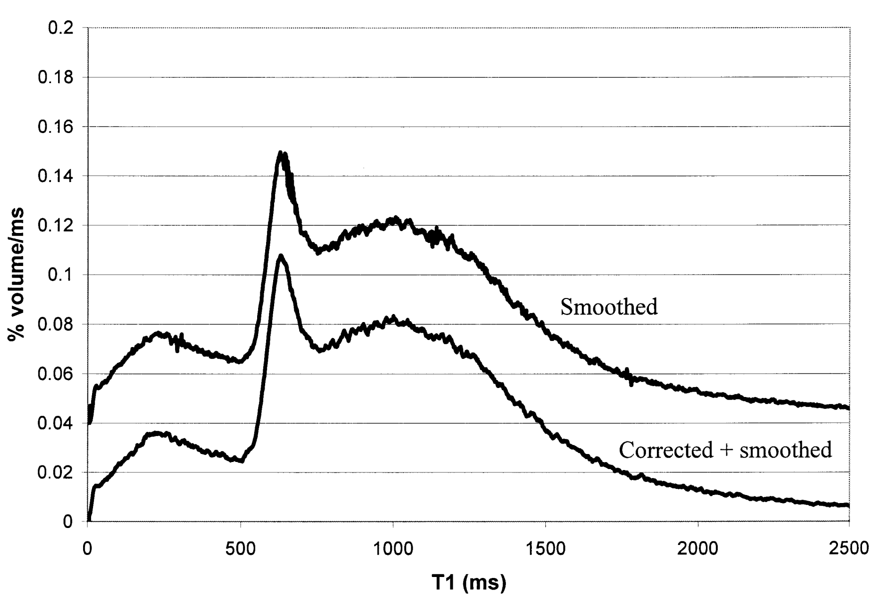

FIG. 4. The lower of these two histograms shows the T_1 histogram after it has been smoothed with a window of width 7 ms and had noise added to the data; the upper curve shows the same histogram, but without the noise correction, displaced slightly. As can be seen, the spikes have been removed on both histograms; however, there are still remnants of these at T_1 values of ~300, 1150, and 1800 ms on the uncorrected image.

while the upper has not. The smoothing has had a clear effect on the histogram. The spikes can still be seen on the histogram derived from uncorrected images, although they are greatly reduced in size. With correction the smoothed histogram has almost no evidence of these spikes. Although there were no significant differences between the two histograms in terms of the metrics described when comparing the noise addition schemes, the remnant spikes could still have an impact on the features extracted, particularly when using methods such as principal component analysis. It also appears that applying the correction has very little effect on subsequent smoothing of the histogram, indicating that there is value in carrying out both procedures.

DISCUSSION

The histograms shown in Fig. 1 are good examples of the type of spikes caused by the uneven distribution of quotients described here. When compared with the histograms derived from corrected images there are some quite large differences. The reduction in peak height in the T_1 histogram, in particular, is quite marked, and if this were repeated across a group of histograms then the peak height obtained from this group would be quite inaccurate. Although less pronounced, the changes in peak location in both histograms could also be significant, particularly, as noted previously, if the true peak is close to the location of an artifactual spike, causing this to be considered the peak.

It can be argued that smoothing is a better way of dealing with these problems; however, the width of the spikes in the T_1 histogram would mean that a smoothing procedure does not completely eliminate the problem, as shown in Fig. 4. Also, the problems that exist with smoothing, some of which were mentioned in the introduction, mean that it is not always an appropriate tool to use. What is useful about this correction is that there appears to be no significant effect on the remainder of the histogram. It seems logical that the addition of noise with a maximum magnitude of 0.5 will produce changes that are smaller than those which are produced by the noise in the MR signal, which is normally on the order of 3–4 signal intensity units for these images on our GE Signa 1.5 T system (General Electric, Milwaukee, WI, USA). From the residual variation between the corrected and uncorrected histograms, it appears that the changes to the histogram brought about by the correction in regions unaffected by spikes are indeed small and should not significantly affect results taken from these regions. It is not surprising that there were no differences seen between the noise addition schemes—the effect of the noise addition on the histograms away from the spikes has been shown to be small, so the variations in them caused by differences in the noise distributions should be smaller yet. This is encouraging, as it means that detailed investigations of which noise addition scheme is most suitable is unnecessary and means that uniformly distributed noise can be used due to its relative ease of implementation; the fact that, unlike normally distributed noise, the range can be limited to ± 0.5, limiting the effect of the noise on the MTR values produced and that it appears to best represent the rounding process within the scanner.

CONCLUSION

It has been shown that any histogram formed from the division of series of integers, either directly or through a lookup table, is susceptible to artifactual spikes caused by the uneven distribution of the quotients of two integers. It has also been shown that the simple addition of noise to the integers before division removes these spikes, whereas the use of smoothing for this is less effective. The technique can be used alongside smoothing to correct for other noise artifacts and does not degrade the signal and characteristics of the original histogram.

ACKNOWLEDGMENTS

The authors thank Dr. Joseph McGowan for useful discussions regarding this work, the MS Society of Great Britain and Northern Ireland for their support of this work, and the NMR research unit.

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