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1981 Phys. Med. Biol. 26 313

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Scientific Note

Definitions of effective energy in computed tomography

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Received 6 June 1980, in final form 25 September 1980

A method of measuring 'effective energy' in CT using 'keV liquids' has recently been suggested (White and Speller 1980). This energy was defined to be the single photon energy at which the linear attenuation coefficient of a particular material equals the linear attenuation coefficient in the polyenergetic beam. This effective energy depends on the material being measured (as pointed out by Brooks (1977)), and the questions arise of how great this dependence is, and how it affects CT numbers calculated using this equivalent energy.

Three distinct effective energies can usefully be considered:

(i) The effective linear attenuation coefficient energy, $E_{\mu,m}$, is the energy at which the linear attenuation coefficient $\mu_m(E)$ of a material m equals that in the polyenergetic beam $\bar{\mu}_m$. $\bar{\mu}_m$ is given by

$$\bar{\mu}_m = \int \mu_m(E) I_d(E) dE \quad (1)$$

where $I_d(E)$ is the normalised detected energy fluence spectrum. $E_{\mu,m}$ is defined by $\mu_m(E_{\mu,m}) = \bar{\mu}_m$ and depends on the material; therefore

$$\mu_m(E_{\mu,m}) = \int \mu_m(E) I_d(E) dE. \quad (2)$$

(ii) The Hounsfield number in a polyenergetic beam, \bar{H}_m , is defined by

$$\bar{H}_m = [(\bar{\mu}_m / \bar{\mu}_w) - 1] 1000 \quad (3)$$

where the w subscript refers to water. For a monoenergetic beam it is

$$H_m(E) = [(\mu_m(E) / \mu_w(E)) - 1] 1000. \quad (4)$$

The effective Hounsfield energy of a material m in a polyenergetic beam, $E_{H,m}$, is now defined as the energy at which the Hounsfield number of the material equals that in the polyenergetic beam:

$$H_m(E_{H,m}) = \bar{H}_m \quad (5)$$

and therefore

$$\frac{\mu_m(E_{H,m})}{\mu_w(E_{H,m})} = \frac{\int \mu_m(E) I_d(E) dE}{\int \mu_w(E) I_d(E) dE}. \quad (6)$$

$E_{H,m}$ depends on the material, and in general $E_{H,m} \neq E_{\mu,m}$, since the former is defined by

the energy dependence of $\mu_m(E)$ and $\mu_w(E)$ (equation (6)), and the latter by $\mu_m(E)$ (equation (2)).

(iii) The effective spectral energy, E_s , is the energy at which the spectral factor equals that of a polyenergetic beam. The spectral factor is the ratio, for water, of all attenuation except Compton scatter to Compton scatter (see Brooks 1977), and is independent of material.

In the appendix it is shown theoretically that $E_{H,m} \approx E_s$, and therefore the effective Hounsfield energy is expected to be approximately independent of material.

In order to answer the original questions, and to test the predicted material independence of $E_{H,m}$, CT spectra were generated by computer and attenuation coefficients calculated, using the method of McCullough (1975), with attenuation values from Hubbell (1969, 1977) and Veigele (1973). Kramer's expression for bremsstrahlung was used, and a detector responding to energy fluence (rather than photon fluence) assumed, with 100% efficiency. Hounsfield numbers were calculated using equations (1), (3) and (4). Beam hardening and other non-linearities were not included. This simulation was not intended to be accurately predictive of Hounsfield numbers or effective energies measured on a real machine, but merely to illustrate, using real attenuation data, some properties of the spectra which are insensitive to its exact shape. The spectra used in tables 1 and 2 represent the extremes of effective

Table 1. 140 kV scan; filtration was 0.35 cm aluminium + 40 cm water; $E_s = 81.1$ keV.

Material	\bar{H}_m (HU)	$E_{\mu,m}$ (keV)	$E_{H,m}$ (keV)	$\Delta H_m(E_{\mu,m})$ (HU)	$\Delta H_m(E_s)$ (HU)	$\Delta H_m(E_{H,CCl_4})$ (HU)
Adipose tissue	-95	89.3	80.8	4	0	0
Water	0	87.9	—	0	0	0
Muscle	33	87.8	81.2	0	0	0
Inner bone	164	85.7	81.0	-10	0	0
Cortical bone	1221	83.5	81.0	-37	-2	2
keV liquid KEV 1/80	0	84.6	80.8	-13	-1	1
Carbon tetrachloride	1253	82.6	80.8	-46	-9	0

Table 2. 100 kV scan; filtration was 0.35 cm aluminium + 24 cm water; $E_s = 60.1$ keV.

Material	\bar{H}_m (HU)	$E_{\mu,m}$ (keV)	$E_{H,m}$ (keV)	$\Delta H_m(E_{\mu,m})$ (HU)	$\Delta H_m(E_s)$ (HU)	$\Delta H_m(E_{H,CCl_4})$ (HU)
Adipose tissue	-118	63.5	59.9	6	0	0
Water	0	62.6	—	0	0	0
Muscle	36	62.5	60.7	0	0	0
Inner bone	244	61.5	60.0	-9	-1	0
Cortical bone	1769	60.8	60.0	-34	-3	-3
keV liquid KEV 1/60	2	61.5	60.1	-8	0	0
Carbon tetrachloride	2241	60.7	60.1	-44	1	0

energies likely to be encountered in practice. The compositions of adipose tissue, muscle and cortical bone were obtained from ICRP Reference Man (1975), and of inner bone and keV liquids from White and Speller (1980).

It can be seen that $E_{\mu,m}$ varies considerably with material (a range of 6 keV for tissue) but that the effective Hounsfield energy $E_{H,m}$ varies little and is close to the

effective spectral energy E_s . The error caused by estimating the Hounsfield number using a single energy E ,

$$\Delta H_m(E) = H_m(E) - \bar{H}_m \quad (7)$$

is shown for

- (i) the effective linear attenuation coefficient energy $E_{\mu,m}$,
- (ii) the effective spectral energy E_s , and

(iii) the effective Hounsfield energy for carbon tetrachloride E_{H,CCl_4} . (Carbon tetrachloride has a large photoelectric contribution, and hence a rapidly varying $H_m(E)$, and is therefore a suitable material with which to estimate the effective spectral energy.)

Use of the effective linear attenuation coefficient energy is clearly inadequate, producing errors up to 37 Hounsfield units (HU) for tissue (compared to a precision in measuring average Hounsfield number over an area of a few square centimetres which can be less than 1 HU). Use of the effective spectral energy, or an estimate of it from carbon tetrachloride, is considerably better, producing a maximum error of 3 HU for tissue.

A real scanner produces a CT number N_m , for a material m , which is linearly related to attenuation coefficient by

$$N_m = a + b\bar{\mu}_m \quad (8)$$

where a and b depend on the machine settings and spectrum. (Beam hardening effects have been ignored.) Calibration of the machine by measuring the CT numbers of air (N_a) and water (N_w), as described by Brooks (1977), gives values for a and b , whence

$$\bar{\mu}_m = [(N_m - N_a)/(N_w - N_a)]\bar{\mu}_w \quad (9)$$

and

$$\bar{H}_m = 1000[(N_m - N_w)/(N_w - N_a)]. \quad (10)$$

Note the distinction here between Hounsfield numbers (corrected for air and water) and CT numbers (uncorrected). The effective energy measured by any method based on Hounsfield numbers will be the effective Hounsfield energy for that material (approximately equal to the effective spectral energy of the beam). The keV liquid system is one such method, since the CT numbers are compared with water, as in equation (10). Measurements of an effective energy have been made (e.g. Rutherford *et al* 1976, Millner *et al* 1978) by finding which single energy E_{eff} gives a linear relationship between calculated linear attenuation coefficient at E_{eff} and CT number for a range of materials. From equations (5) and (4),

$$\bar{H}_m = H_m(E_{H,m}) \approx H_m(E_s) = [(\mu_m(E_s)/\mu_w(E_s)) - 1] 1000 \quad (11)$$

and therefore

$$N_m = N_a + [(N_w - N_a)/\mu_w(E_s)]\mu_m(E_s). \quad (12)$$

This is linear in $\mu_m(E_{\text{eff}})$ if $E_{\text{eff}} = E_s$, and therefore measurements of this type also produce an estimate of effective spectral energy. Note that only a single material, having a large photoelectric contribution, is required (in addition to air and water) to estimate the effective spectral energy. The 'linearity' equation described by White and Speller (1980) is then given by equation (12).

McCullough (1975) defined six effective energies, three of which are the effective linear attenuation coefficient energies of water, for a detector which responds to photon

fluence, energy fluence (as in the present simulation), or exposure. These are all significantly different from the effective Hounsfield energy of the materials under consideration.

Most scanners have an amount of detected beam filtration which varies with position across the projection (and depends on angle if the object is not radially symmetric). As a result the effective energies depend on the object being scanned, and on position within the object. Unless this has been software-corrected, the effective spectral energy may need to be measured at a variety of positions within the object.

In conclusion, the effective Hounsfield energy has been newly defined, and shown to be approximately material-independent and equal to the effective spectral energy. Existing methods of measuring the effective energy of a polyenergetic beam produce estimates of the effective spectral energy, and this is also the energy required to predict or interpret CT numbers.

Appendix

Brooks (1977) has shown that for the energies and elements of interest in CT the linear attenuation coefficient of a material may be expressed as the sum of two components, each the product of a material-dependent (but approximately energy-independent) factor and an energy-dependent factor:

$$\mu_m(E) = f_{C,m}\mu_{C,w}(E) + f_{p,m}\mu_{p,w}(E). \quad (13)$$

The first term refers to Compton attenuation, and the second to the remaining attenuation (mainly photoelectric). For water $f_{C,w} = f_{C,p} = 1$.

The spectral factor for a polyenergetic beam is

$$\bar{\beta} = \frac{\int \mu_{p,w}(E)I_d(E) dE}{\int \mu_{C,w}(E)I_d(E) dE}. \quad (14)$$

For a monoenergetic beam this becomes

$$\beta(E) = \mu_{p,w}(E)/\mu_{C,w}(E). \quad (15)$$

Since $\beta(E_s) = \bar{\beta}$, we have

$$\beta(E_s) = \frac{\int \mu_{p,w}(E)I_d(E) dE}{\int \mu_{C,w}(E)I_d(E) dE}. \quad (16)$$

Substituting equation (13) for $\mu_m(E)$ into equation (6), and using equations (15) and (16), yields

$$\frac{f_{C,m} + \beta(E_{H,m})f_{p,m}}{1 + \beta(E_{H,m})} = \frac{f_{C,m} + \beta(E_s)f_{p,m}}{1 + \beta(E_s)}. \quad (17)$$

Therefore $E_{H,m} = E_s$ (within the limitations of equation (13)).

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